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Research on complexity evolution of marketing evaluation data based on fractional calculus

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A B S T R A C T

Firstly, the paper introduces the definition of Fourier transform of fractional calculus equation, and then clarifies the discretization algorithm of fractional calculus Fourier. Secondly, the Fourier transform is applied to the processing of marketing data signals, including market signal detection and parameter estimation, signal de-drying and so on. The paper finds that the discretization algorithm of fractional calculus Fourier has strong theoretical and practical significance in the detection and complexity evolution of marketing evaluation data.

1. Introduction

Fractional Fourier Transform (FrFT) attention of many researchers because of their nature do not have many of the traditional Fourier transform, and are used in many scientific and engineering technology. The quantum mechanics [1], an optical system and an optical signal processing [2], light image processing [3]. FrFT reason was first applied in the optical signal processing is relatively easy to achieve because the optical, not until the mid-1990s, since the discretization method is proposed more fractional Fourier transform and fast algorithm, such that it FrFT truly reflected in the field of electrical signal processing in its value [4]. When the current study, FrFT and short-time Fourier transform, wavelet transform, Wigner distribution, Radon-Wigner transform frequency analysis tool of internal relations, improve some method for non-stationary signals, and further expand the application fields of FrFT. The Radon-Wigner transform is often used for a variety of time-varying signal analysis, and the relation between the Radon-Wigner transform FrFT show: the modular square signal FrFT exactly Radon-Wigner transformation in that direction. Based on this relationship, many studies Radon-Wigner transform can be applied directly to FrFT in. Further, in some cases, be replaced with other FrFT frequency transform may also bring some advantages, such as FrFT be implemented by means of FFT, the calculation method is simple; on the other hand, FrFT is one-dimensional linear transform, the multisession the amount of signal conditions, can effectively avoid the interference cross term [5].

Currently fractional calculus Fourier transform as a new class of signal analysis tools, with a very wide range of applications in the field of signal processing. In recent years, new research results are emerging. Be analyzed from the perspective of treatment, the current applied research FrFT at home and abroad mainly around the following ideas:

Firstly, the use of the focusing of FrFT. Some theoretical and applied directly to the traditional Fourier transform generalized to fractional Fourier transform domain. Commonly used in conventional Fourier transform stationary signal analysis and processing, for non-stationary signals, the signal analysis processing capability is varying failure, and the type of signal FrFT exhibit good analysis. Traditional Fourier transform can be understood as a signal developed on a complete set of orthogonal sine, so Fourier transform signal is a sinusoidal impulse function; on Fractional Fourier Transform is to be understood that a signal is developed on a set of orthogonal Chirp group of, corresponding, Chirp signal in a specific order of the FrFT also an impulse function. Focusing analysis and processing of the class Chirp signal is very favorable, is applied directly to the Chirp signal detection and parameter estimation. It is widely used because Chirp signal communication, sonar, biomedical, and the like, especially in the modern radar system is therefore

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used to target detection and tracking multiple radar signal processing, analysis and treatment of SAR and ISAR algorithm based FrFT imaged, the motion parameter estimation techniques [6].

Secondly, when using the rotation frequency characteristics FrFT. A Wigner distribution FrFT rotation signal is the coordinate of the Wigner distribution form the original signal. This characteristic when the rotational frequency FrFT for non-stationary signal analysis and processing is very favorable. In practical engineering applications, and noise suppression to extract useful signal is a very important issue. Conventional filtering methods generally limited to the frequency-domain windowing or Masking processing, but when there is strong when the frequency coupling between the signal and noise, the conventional signal to noise filter cannot effectively achieve the separation. At this time, the coordinate axis FrFT to an appropriate angle, decouple the signal and noise on the new fractional Fourier transform domain, can be achieved without distortion and recover completely filtering noise signals. This is the basic principle of fractional Fourier domain filtering [7].

Herein is primarily utilized fractional Fourier transform domain signal comprising a signal detection and parameter estimation market, signals and the like to dry, full use of the principle of Fourier domain filtering.

2. Fractional Fourier Transform definition

Fractional Fourier Transform yet angle Fourier transform (AFT) or rotated Fourier transform (RFT), FrFT define the function $x(t)$ is as follows [8]:

$$X_a(u) = F^a[x(t)] = \int_{-\infty}^{\infty} x(t)K_a(t,u)dt \tag{1}$$

Let $A_\varphi = \sqrt{1 - j\cot\varphi}$, $n = 1, 2, \ldots$, then the kernel function

$$K_a(t,u) = \begin{cases} A_\varphi \exp(j\pi(t^2+u^2)\cot\varphi - j2\pi ut\csc\varphi) & \varphi \neq n\pi \\ \delta(t-u) & \varphi = 2n\pi \\ \delta(t+u) & \varphi = (2n+1)\pi \end{cases} \tag{2}$$

Where $\varphi = a\frac{\pi}{2}$ is the rotation angle of the time-frequency plane, a is the order of FrFT, F^a is the FrFT operator, and $\delta(t)$ is the unit pulse function. It can be found that FrFT takes 4 cycles, if and only if $a = 4n$ (i.e. $\varphi = 2n\pi$), FrFT results in $x(t)$, G 1 (i.e. $\varphi = 2n\pi + \frac{\pi}{2}$), FrFT is Fourier transform, $a = 4n+ 2$ (i.e. $\varphi = 2n\pi + \pi$), FrFT When the result is $x(-t)$, $a = 4n+ 3$ (i.e., $\varphi = 2n\pi + \frac{3\pi}{2}$), the result of FrFT is a negative Fourier transform. Therefore, only a fractional Fourier transform is performed on $a \in (0, 2)$.

The formula (1) and (2), FrFT definition equation rewritten as:

$$X_a(u) = A_\varphi \int_{-\infty}^{\infty} x(t)\exp\left[j\pi(u^2+t^2)\cot\varphi - j2\pi ut\csc\varphi\right]dt, \varphi \neq n\pi \tag{3}$$

3. Fractional Fourier Transform discretization

In order to facilitate computer processing must be discretized kernel function of the input signal and FrFT from the basic definition of which can be seen FrFT discrete computationally complex than the DFT, and therefore, a highly accurate and fast algorithm FrFT discretization It is the urgent problem of engineering applications. None of the many discrete methods proposed by scholars at home and abroad can satisfy all of the above requirements at the same time. Currently, there are three main feasible discretization method [9]: Bian-like discrete type, wherein the decomposition-type, linear-weighted. Table 1 shows the comparative results of the three DFRFT.

DFRFT various algorithms currently available, the discrete sampling type converting DFRFT continuous approximation, high accuracy, low computational complexity, and with a small amount of computation expressions of the form is closed, it is widely used non-uniform domain scores preclude comp and reconstruction, Chirp signal detection and parameter estimation, scores research domain filtering, and the like, is currently the most widely used numerical method.

The algorithm proposed by Ozaktas [10], also known decomposition method: starting from continuous FrFT (1) is defined, firstly decompose the complex integral expression, reduced to a few simple calculations and then through discretization Finally, the resulting discrete convolution of its expression, which can be calculated using an FFT discrete convolution, so this introduces performs sampling of the method. Dimensional normalization process is as follows:

Provided a time domain interval of the signal $[-\frac{T}{2}, \frac{T}{2}]$, the frequency-domain interval $[-\frac{F}{2}, \frac{F}{2}]$. The time domain and frequency domain are transformed into the same dimension field, introducing a dimensionless normalization factor S, i.e.,

$$S = \left(\frac{T}{F}\right)^{\frac{1}{2}} \tag{4}$$

Where T is the time width of the signal and F is the bandwidth of the signal.

Then the dimension normalized coordinates are:

$$t' = \frac{t}{S} \qquad f' = fS \tag{5}$$

(t', f') new coordinate system to achieve the dimensionless normalized rear section $[-\frac{\Delta x}{2}, \frac{\Delta x}{2}]$ are time domain and frequency-domain normalized, where $\Delta x = (TF)^{\frac{1}{2}}$ is the sampling frequency, the sampling interval becomes the signal $\frac{1}{\Delta x}$.

Dimensional Normalization of a signal can be sampled type DFRFT, the specific steps are as follows [11]:

Step 1: the signal $x(t)$ is multiplied by chirp signal $\exp(-j\pi t^2 \tan\frac{\varphi}{2})$, i.e.,

$$g(t) = \exp(-j\pi t^2 \tan\frac{\varphi}{2})x(t) \tag{6}$$

Step 2: $g(t)\exp(j\pi t^2 \csc\varphi)$ convolve the chirp signal, i.e.,

$$h(u) = A_\varphi \int_{-\infty}^{\infty} \exp\left[j\pi\csc\varphi(u-t)^2 g(t)\right]dt \tag{7}$$

Table 1
Comparison of three DFRFT results.

Method	Superiority	Disadvantaged	Nature	Completion of calculation
Discrete Bian like type	Direct application of Shannon sampling theorem	Lose some important properties	Approximate continuity and having a closed form	$O(N\log N)$
Wherein decomposition type	Simple implementation of Fractional Fourier operator	No closed form, is not conducive to real-time processing	Having sum and approximate continuity rotation	$O(N^2)$
Weighted linear type	It has some important properties of FrFT	The results of the continuous errors greater FrFT	And having a rotary additivity closed form	$O(N\log N)$

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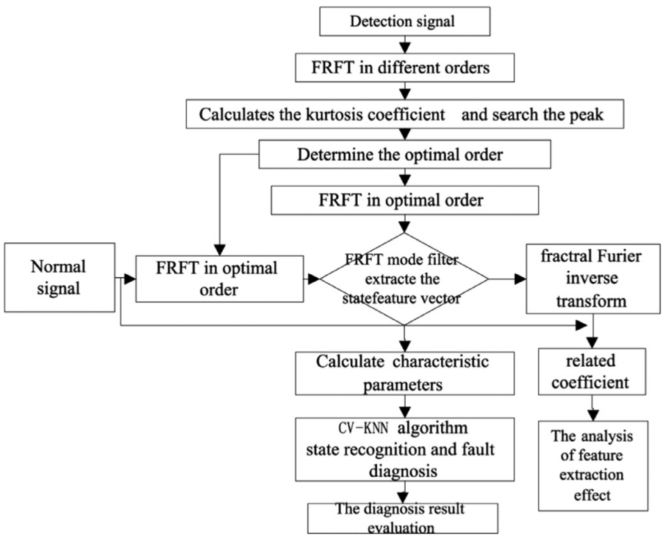

Fig. 1. A-order FrFT flow chart.

Step 3: $\exp(-j\pi u^2 \tan\frac{\varphi}{2})$ chirp signal is multiplied with the signal $h(u)$, i.e.,

$$X_a(u) = \exp(-j\pi u^2 \tan\frac{\varphi}{2})h(u) \quad (8)$$

DFRFT can be implemented by means of FFT, the flow chart shown in Fig. 1.

4. Application of FrFT in marketing data signal processing

Since the fractional Fourier transform may be interpreted as Chirp decomposed, fractional Fourier transform is particularly suitable for processing class Chirp signal. Marketing common signal is the market demand as well as market-related signal, because it presents a different letter signals gathered in different domains of FrFT the order shown in Figs. 2 and 3, by this feature in the marketing system be FrFT domain peak two-dimensional search, you can achieve Chirp signal detection and parameter estimation.

Most studies [12] Currently, Chirp signal is detected and the parameters are estimated based on this idea, the system block diagram shown in Fig. 4.

Let $x(t)$ in Fig. 3 be the LMF signal to be detected, i.e.

$$x(t) = \exp\left[j(2\pi f_0 t + \pi k t^2)\right] + n(t), -\frac{T}{2} \le t \le \frac{T}{2} \quad (9)$$

Wherein, f_0 represents an initial frequency data signal, k represents a marketing data modulation frequency, $n(t)$ denotes Gaussian noise.

By its dimension after normalization on Fractional Fourier Transform, which give two-dimensional distribution diagram shown in Fig. 5.

As it can be seen from Fig. 4, when the order of 1.156 a, data aggregation FrFT maximum value, at this time the rotation angle $\hat{\varphi} = 1.156 \times \frac{\pi}{2} = 0.578\pi$, $|X_a(u)|^2$ corresponding to the maximum $\hat{u}_0 = 4.85$, in accordance with formula (9) when the initial parameter can be estimated frequency modulation frequencies $\hat{f}_0$ and $\hat{k}$:

$$\begin{cases} \hat{f}_0 = \hat{u}_0 \csc \hat{\varphi}_0 \\ \hat{k} = -\cot \hat{\varphi}_0 \end{cases} \quad (10)$$

By the above calculation, and the parameters can be done to detect Chirp signal estimate that at low SNR background Gaussian noise, which still exhibits very good test results.

Under normal circumstances, we cannot accurately predict random signal in the current implementation of the value of a moment, but, generally subject to random signal to determine the probability distribution and the joint probability distribution. To marketing data, sales data, the value of its sales data signal has a probability distribution and probability density function determined. In addition, after several actual observation and statistics, can clearly grasp the statistical characteristics which determine the amount of sales data have many of the properties of these statistical features can reflect signals of these properties is the basis for effective market trends and future production evaluation. In the marketing system, a point noise may cause some false positives production plan, therefore, in order to obtain a clear and accurate marketing sales data signal to improve the accuracy of analysis and diagnosis, the signal must be some analysis and processing, so that the data curve smoother, more prominent feature point.

If the signal noise superimposed projected on the time axis (shown in Fig. 6) does not exist, a suitable filter may be employed to filter out noise in the time domain; if there is no noise in the signal and the frequency axis projection overlap, it can be filtered through a suitable filter interference in the frequency domain at this time; however, when the interference signal and noise are present in the projection overlap time domain and frequency domain, frequency coupling (i.e. the presence of Fig. 6), when this time is not possible only by complete domain or frequency domain filtering to filter out noise. However, fractional Fourier transform coordinates may be rotated to a certain angle, frequency coupling when released, to maximize filter noise. That is, the fractional Fourier domain a certain angle can be better separation of signal and noise. Moreover, time domain and frequency domain may

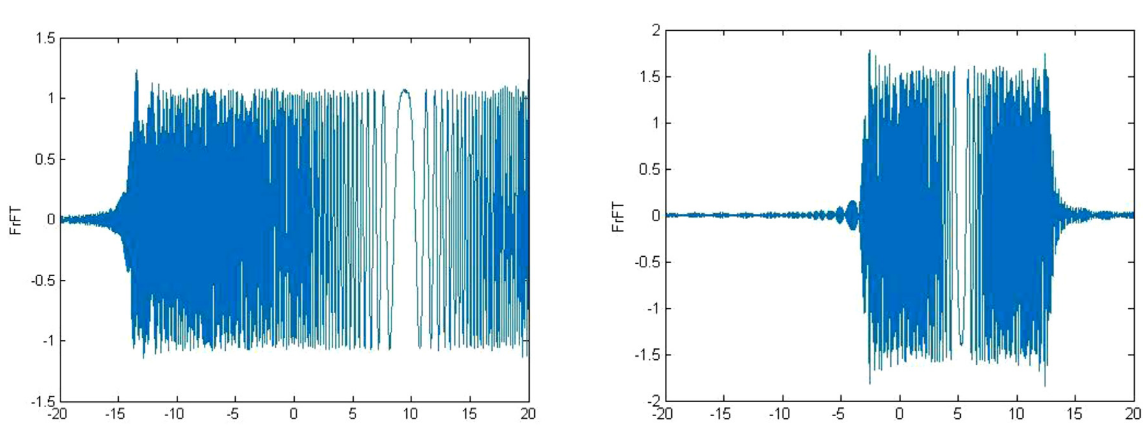

Fig. 2. Signal aggregation of the marketing system FrFT when a= 0.5 and a= 0.9.

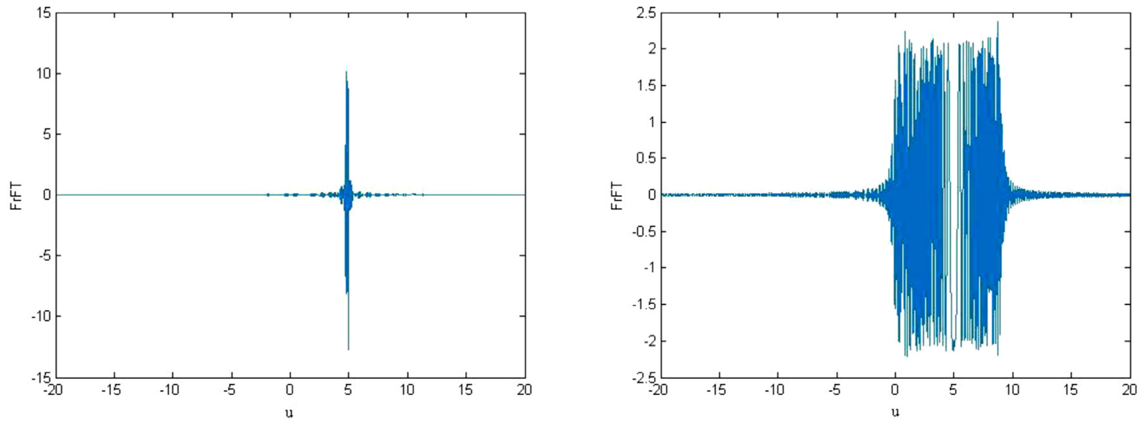

Fig. 3. Signal aggregation of the marketing system FrFT when a= 1.16 and a= 1.3.

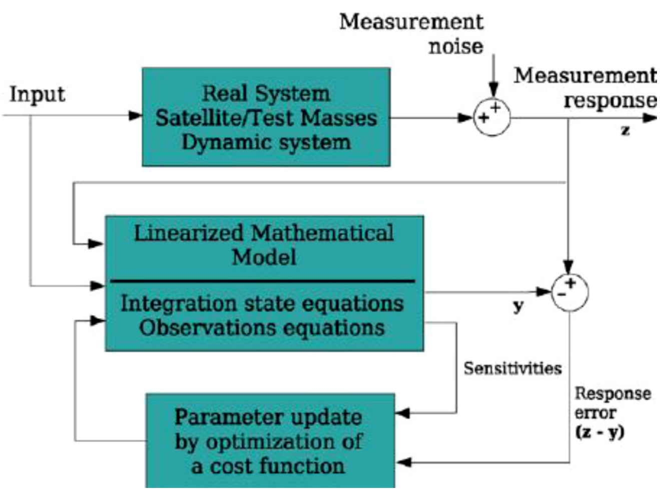

Fig. 4. Chirp signal parameter estimation system block diagram.

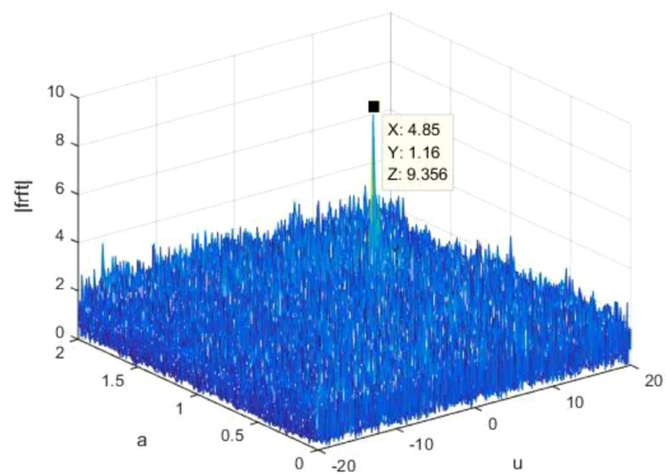

Fig. 5. Two-dimensional map of FrFT.

also be seen as fractional Fourier transform domain, in special circumstances, when changing the order of a= 0, corresponding to the time domain, when changing the order be a= 1, corresponding frequency domain, as shown in Fig. 6 shown.

Thus, can be utilized in a fractional Fourier domain signal separation and noise characteristics of the marketing data signal on Fractional Fourier Transform, to find that the optimum rotation angle of the signal and noise separation where the Fractional Fourier domain filtering, and then to subjected to fractional Fourier inverse transform, to give the final denoised signal specific algorithm shown in Fig. 7.

Through the above analysis, the specific steps of the algorithm are as follows [13]:

(1) The FrFT symmetry, converting the order $a \in [0, 2)$, $x(n)$ seeking the input signal FrFT;
(2) The signal after transform two-dimensional search to find the best conversion order of a';
(3) Calculate the signal $x(n)$ FrFT a' is at the order of;
(4) Filtering the optimum fractional Fourier transform domain;
(5) The filtered signal fractional Fourier inverse transform, i.e. $-a'$ stage on Fractional Fourier Transform, to obtain time domain output signal $y(n)$.

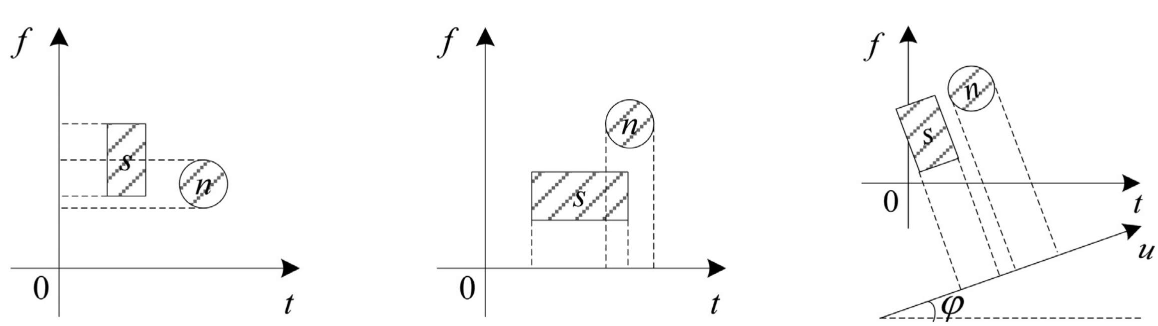

Fig. 6. Time domain filtering $a = 0$, $a = 1$ and Fractional Fourier domain filtering.

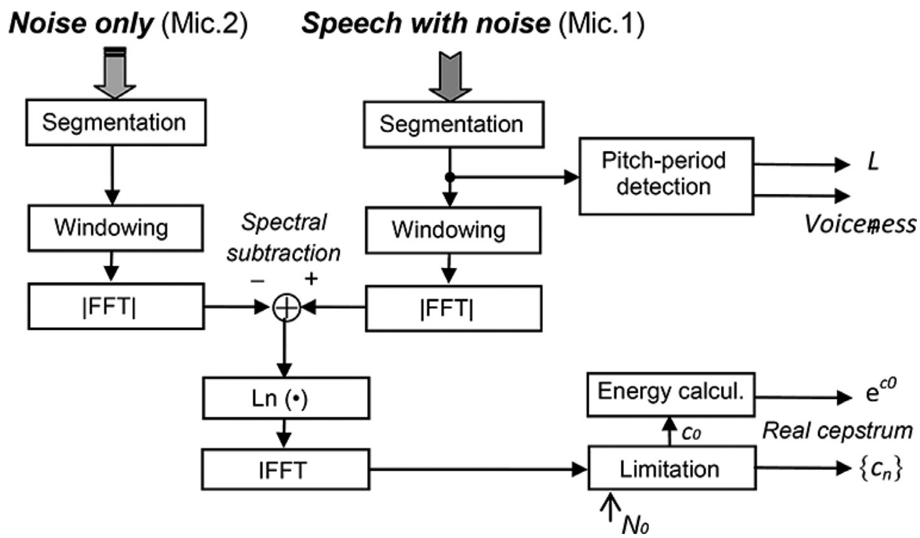

Fig. 7. Block diagram of the denoising algorithm.

5. Conclusion

Frequency transform paper can be seen from the relationship between the transform domain and time domain, frequency domain fractional Fourier, fractional Fourier transform is essentially a unified, capable of simultaneously reflecting marketing data signals in the time domain, frequency domain Information. Several applications described herein Fractional Fourier transformation by signal processing in the art may find that it is suitable for non-stationary signals such as Chirp class, and because more than one transformation parameter order of freedom, so the Fractional Fourier Transform under certain conditions the effect of the frequency distribution tends to be obtained or not when the Fourier transform traditional, since it has a relatively fast discrete mature algorithm, to obtain better results in the same time does not need to pay too much computational cost, it has a very wide range of applications engineering.

Declaration of Competing Interest

None.

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